Defining Path Cost for a Quantum Network

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Abstract

To date, research on entangled quantum networks has primarily focused on an abstract model consisting of a linear chain of repeaters, with a power of two number of hops of identical length and quality. We are analyzing the behavior of more complex network topologies. In a network of heterogeneous links and irregular topology, path selection affects both the performance of individual connection and global network load. We propose a definition for link cost and an algorithm for ranking candidate paths to maximize the throughput of end-to-end connections, measured in high-fidelity Bell pairs per second. Simulations confirm reasonable agreement between the calculated path cost and the expected throughput. Thus, we propose the use of a quantum form of Dijkstra's algorithm[1].

Problem

Networks of quantum repeaters use purification and entanglement swapping to create high-fidelity end-to-end Bell pairs which are then used in distributed quantum applications. For homogeneous paths of 2^n hops quantum repeaters operate well using a simple doubling of distance for each entanglement swap. In real-world networks, candidate paths will have differing numbers of hops of varying quality. On such heterogeneous paths, we must reconsider when and where entanglement swapping takes place, and devise an algorithm for ranking paths.

Approach

We wish to define a cost for each path, allowing us to order a set of paths so that low-cost paths better meet out goals for the overall system. Here, we choose maximizing throughput as our goal, meaning that high-bandwidth paths (measured in high-fidelity Bell pairs per second) will have a low cost. To achieve this, we need both a way to establish the cost for a single link, and a way to calculate path cost from a set of link costs. We use the inverse of single-hop throughput as our link cost. Path cost, as in Dijkstra's algorithm, is the sum of the link costs in the path. We have extended our quantum network simulator to calculate the throughput of end-to-end connections for heterogeneous links. Non-power of two lengths and path-adapted entanglement swapping remain as future work. We expect to use both static and dynamically adaptive methods.

Results

In our simulation, we used three types of qubus-based repeater links (see [2] and references therein).

- Good link (◯)
  - throughput: 835.4 Bell pairs/sec
  - Link cost: 1
- Fair link (△)
  - throughput: 419.4 Bell pairs/sec
  - Link cost: 1.70

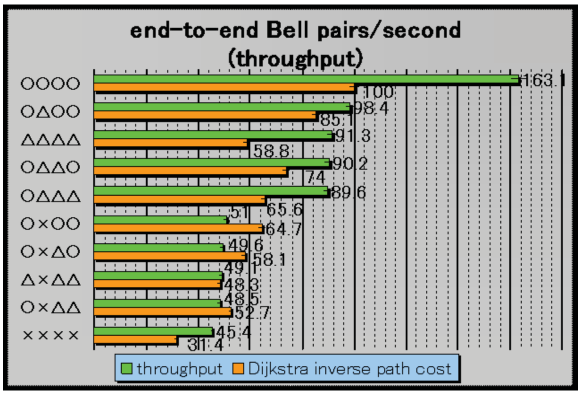

Figure 1: Throughput and the inverse of path cost for four-hop paths. ◯, △ and × represent good, fair, and poor links, respectively.

- Poor link (×)
  - throughput: 263.0 Bell pairs/sec
  - Link cost: 3.18

We tested ten types of end-to-end connections and compared simulated throughput and Dijkstra's cost (plotted as its inverse for ease of comparison), as shown in Fig. 1. Our simulations suggest that, as in classical networks, the throughput of a single path will approach the throughput of the lowest-quality link. As in classical networks, Dijkstra's algorithm provides reasonable but not perfect ordering of candidate paths, and should be useful in the real world.

References

[1] EW Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematik*, Vol. 1, No. 1, pp. 269–271, 1959.

[2] Rodney Van Meter, Thaddeus D. Ladd, W. J. Munro, and Kae Nemoto. System design for a long-line quantum repeater. Vol. 17, No. 3, pp. 1002–1013, June 2009.